Crypto-Synthetic Market Making

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Abstract

Cap is a crypto-synthetic trading protocol that uses a price feed and a liquidity pool to make markets. This technical paper describes how Cap's automated market maker ("AMM") works. The AMM needs to provide liquidity in an optimal way so as to minimize risks tied to market volatility, inventory, and informed traders while maximizing order executions.

1 Introduction

Crypto-synthetics are cryptocurrency backed instruments that track the value of an underlying asset (e.g. a stock). They allow investors to gain exposure to an asset without needing to own it or interacting with the traditional financial system, saving time and money. To set up a crypto-synthetic market, both a price feed and a market maker are needed. This paper describes the automated market maker that runs on Cap, a crypto-synthetic trading protocol [1].

An automated market maker is a hands-off liquidity provider. It proposes prices at which it stands ready to buy or sell an asset, while trying to remain as neutral as possible with regards to market direction. Market makers therefore face a trade-off: provide too much liquidity and risk holding excess inventory that can lead to losses, or provide too little liquidity and risk executing too few trades, missing out on revenue [2]. Market makers also face the asymmetric information risk arising from informed traders. An effective market maker must adapt their quotes dynamically to mitigate these risks.

In this paper we propose a zero-intelligence automated market maker ("AMM") for crypto-synthetics. It can provide liquidity for any asset with a price feed, including stocks, ETFs, cryptos, bonds, commodities, foreign exchange, and more. It is conflict-free, transparent, and aims to outperform underlying spot liquidity in most market conditions. The quoted liquidity must strike a balance between protecting the AMM from market risk and providing traders with superior order execution quality.

The model we propose adapts quoted liquidity based on price volatility and the AMM's inventory, the two sources of risk the AMM faces [3]. We begin by introducing the model's framework, then we explore the impact of volatility and inventory on liquidity distribution to obtain a liquidity curve formula.

We also propose a formula for the reservation price, which is the price around which liquidity is quoted. We then walk through an algorithm for market order execution and finally apply our findings through an example on BTC/USD.

2 Framework

In our model we assume that our AMM is the monopolistic (sole) player in the market and the "true" price of the asset is given by the market mid price. We also assume that traders can open both long and short positions, so our AMM can hold both long and short positions in its inventory.

We want to design an automated market maker that can provide a maximum amount of liquidity to traders while minimizing risks tied to volatility and inventory. We are also looking to emulate order book properties — depth, incremental execution, and transparency — without the inefficiencies. At any given time, our model therefore needs to determine how much liquidity to provide and how that liquidity is distributed around the mid price.

For this we use **liquidity curves**, which are Gaussian functions whose properties — amplitude, center, and standard deviation — are automatically adjusted based on volatility and inventory. Liquidity curves are quoted on each side of a **reservation price**, which we define below.

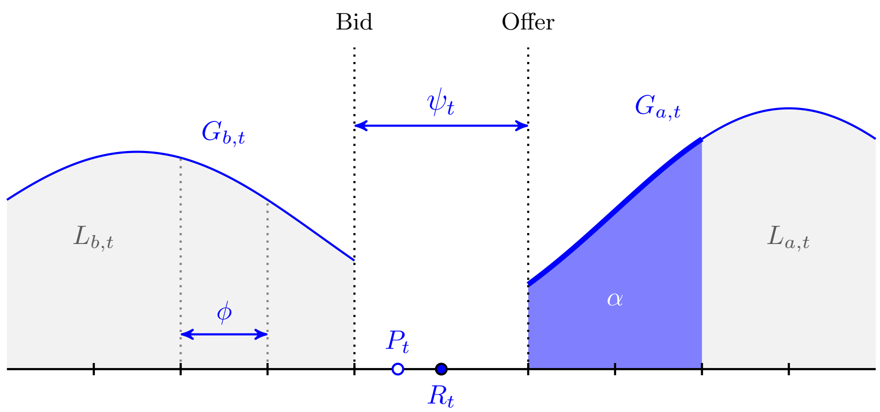

Figure 1: Model Framework

Notations:

- $\mathbf{P_t}$. The market's mid price, received from the price feed.
- $\mathbf{R_t}$. The reservation price around which liquidity curves are quoted. This is equal to the mid price plus an inventory-dependent skew.[1]

[1]See Section 4 for details.

- $\sigma_\mathbf{t}$. Measure of price volatility equal to a k-period standard deviation.
- $\mathbf{Q_t}$. The AMM's inventory for this market, equal to the size of the AMM's net long/short position.
- $\psi_\mathbf{t}$. The base spread inside which no liquidity is quoted. It is centered around R_t and equal to $\gamma_\psi \sigma_t$ where γ_ψ is a risk aversion factor.
- **BBO**. The best bid or offer currently quoted by our model. This is equal to $R_t \pm \psi_t$.
- ϕ. The market's tick size. A tick size of $0.10 means 10 ticks is equal to $1. Ticks are integers, enabling normalized calculations across markets.
- $\mathbf{G_{a|b,t}}$. The liquidity curve function at time t on the ask (bid) side.
- $\mathbf{L_{a|b,t}}$. The total liquidity available at time t on the ask (bid) side.
- $\mathbf{L_{total}}$. Initial amount of liquidity offered on each side of the price. This is manually set for each market.
- α. An incoming market buy order consuming liquidity on the curve.

In a nutshell, our model quotes liquidity curves around a reservation price. In the next two sections, we obtain formulas for G_t, L_t, and R_t, which allows us to fully model the behavior of our AMM.

3 Liquidity Curve

The liquidity curve is the main building block of our AMM. Modeling it with a Gaussian function allows us to offer incremental liquidity at several price levels, concentrate most liquidity around a given price, and enable better execution for smaller orders as compared to a static bid-offer dealer.

We begin by defining a standard Gaussian function to model liquidity provided at time t at a distance of n ticks from the BBO:

$$G_t(n) = a_t \exp(-\frac{1}{2}(\frac{n - b_t}{c_t})^2)$$

where

- a_t is the height of the curve's peak at time t
- b_t is the position of the center of the peak at time t
- c_t is the standard deviation, controlling the width of the curve, at time t

The area under the curve is the total liquidity offered at time t, L_t, so

$$L_t = \int_{-\infty}^{\infty} G_t(n)dn$$

Since G_t is a Gaussian function, we know that

$$\int_{-\infty}^{\infty} G_t(n)dn = a_t c_t \cdot \sqrt{2\pi}$$

So

$$L_t = a_t c_t \cdot \sqrt{2\pi}$$

hence

$$a_t = \frac{L_t}{c_t\sqrt{2\pi}}$$

Therefore

$$G_t(n) = \frac{L_t}{c_t\sqrt{2\pi}} \exp\left(-\frac{(n-b_t)^2}{2c_t^2}\right) \tag{1}$$

Below we find formulas for b_t, c_t, and L_t to obtain the general form of our liquidity curve formula.

3.1 Volatility Impact

Higher price volatility means lower liquidity and increased uncertainty. This implies that our liquidity curve should be centered further away from price and it should be more dispersed ("flatter") as volatility increases.

b_t determines the distance of the center of our liquidity curve from the reservation price. b_t should grow linearly with volatility, so

$$b_t = \gamma_v \sigma_t$$

where γ_v is a risk aversion factor.

c_t controls the width of our liquidity curve. A higher c_t results in a flatter curve and c_t should also grow linearly with volatility. So

$$c_t = \gamma_d \sigma_t$$

where γ_d is a risk aversion factor.

Higher values for γ_v and γ_d lead to a more significant reaction to price volatility by our AMM e.g. an increased risk aversion.

By replacing b_t and c_t with their formulas in (1), we obtain:

$$G_t(n) = \frac{L_t}{\gamma_d\sigma_t\sqrt{2\pi}} \exp\left(-\frac{(n-\gamma_v\sigma_t)^2}{2\gamma_d^2\sigma_t^2}\right) \tag{2}$$

In Figure 2, we plot G_t for several values of σ. We set $\gamma_d = 0.25$, $\gamma_v = 1$, and $L_t = 40\sqrt{2\pi}$. Intuitively $\gamma_v = 4\gamma_d$ means volatility has four times of an effect on liquidity distance from the reservation price than on liquidity dispersion.

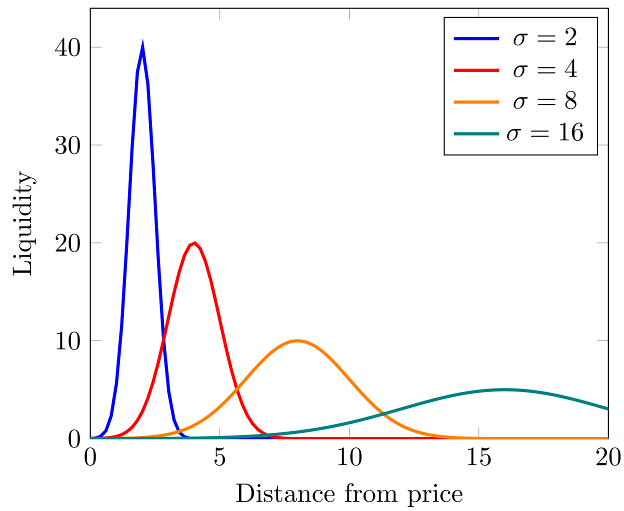

Figure 2: Evolution of Liquidity Curves with σ

3.2 Inventory Impact

The amount of liquidity offered by our AMM at time t, L_t, is a function of the initial liquidity offered, plus an inventory-dependent skew. A net long AMM inventory, for example, should lead our AMM to quote less liquidity on the bid side and more liquidity on the ask side, in an effort to sell excess inventory and return to a market-neutral position. So we have

$$L_t = L_{total} + \Delta_q(t)$$

where $\Delta_q(t)$ is the liquidity drag or boost at time t that compensates for the AMM's inventory imbalance.

- If $Q_t = 0$ then $\Delta_q(t) = 0$.
- If $Q_t > 0$, the AMM is net long and looking to sell their long position, so bid-side liquidity should decrease and ask-side liquidity should increase.
- If $Q_t < 0$, the AMM is net short and looking to buy back their short position, so bid-side liquidity should increase and ask-side liquidity should decrease.

Therefore

$$\Delta_{q,b}(t) = -\gamma_q Q_t$$

$$\Delta_{q,a}(t) = \gamma_q Q_t$$

or more simply

$$\Delta_q(t) = \pm\gamma_q Q_t$$

where γ_q is a scaling factor that determines how much of an effect inventory imbalance has on liquidity drag or boost. To avoid illiquid books, we set a maximum liquidity drag or boost of $\frac{L_{total}}{2}$.

In Figure 3, Δ_q is expressed as a percentage of L_{total} on the ask side of the market.

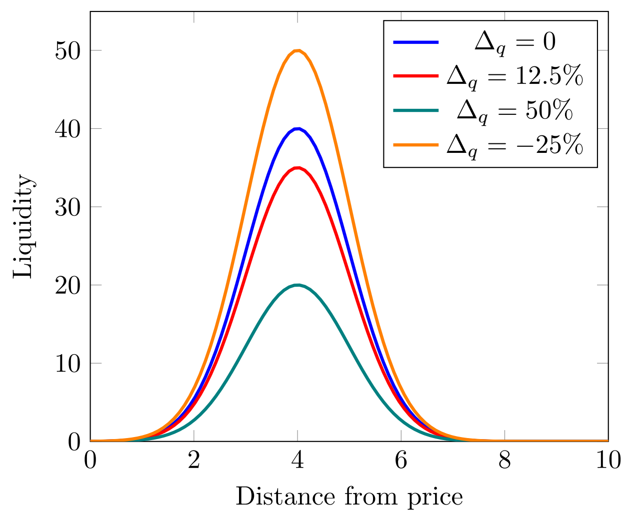

Figure 3: Evolution of Liquidity Curves with Δ_q

3.3 General form liquidity curve formulas

By replacing L_t with its formula in (2), we obtain the general form liquidity curve formula for each side of the market.

$$\boxed{G_{b,t}(n) = \frac{L_{total} - \gamma_q Q_t}{\gamma_d \sigma_t \sqrt{2\pi}} \exp\left(-\frac{(n - \gamma_v \sigma_t)^2}{2\gamma_d^2 \sigma_t^2}\right)}$$

$$\boxed{G_{a,t}(n) = \frac{L_{total} + \gamma_q Q_t}{\gamma_d \sigma_t \sqrt{2\pi}} \exp\left(-\frac{(n - \gamma_v \sigma_t)^2}{2\gamma_d^2 \sigma_t^2}\right)}$$

4 Reservation Price

The reservation price R_t is the price around which liquidity curves are quoted. This price depends on the AMM's inventory. A net short inventory, for example, should lead our AMM to quote more attractive ask prices in an effort to rid its inventory and return to a market-neutral position. This implies a lower reservation price. So

$$R_t = P_t + \phi \cdot \Delta_p(t)$$

where P_t is the market mid price, $\Delta_p(t)$ is the price skew at time t, and ϕ is the market tick size.

- If $Q_t = 0$ then $\Delta_p(t) = 0$.
- If $Q_t > 0$, the AMM is net long. So bids and asks should be lowered to quote more attractive buy prices to traders. So $\Delta_p(t) < 0$.
- If $Q_t < 0$, the AMM is net short. So bids and asks should be increased to quote more attractive sell prices to traders. So $\Delta_p(t) > 0$.

We therefore have

$$\Delta_p(t) = \frac{\gamma_r Q_t}{L_{total}} \sigma_t$$

where γ_r is a scaling factor that determines how much of an effect inventory imbalance has on the reservation price. We set a maximum price skew of $2\sigma_t$ to avoid a significant detachment from the underlying mid price.

In Figure 4, $L_{total} = 100$. At $Q_t = 200$, $\Delta_p(t)$ reaches its max value of $2\sigma_t$.

5 Market order execution

An incoming order will consume a segment of the area under the curve equal to the size of the order, which is the integral between 0 and a distance m (in ticks) from the BBO. In practice, the liquidity curve is discrete, not continuous, so a market order execution can be modeled by a summation of the liquidity consumed at every price tick until the order is fully consumed. In Figure 5, a market buy order consumes two levels of liquidity fully and a third level partially.

The liquidity consumed by a market order of size α is

$$\sum_{n=0}^{m} G_t(n) = \alpha$$

Which is equivalent to

$$\frac{L_{total} \pm \gamma_q Q_t}{\gamma_d \sigma_t \sqrt{2\pi}} \sum_{n=0}^{m} \exp\left(-\frac{(n - \gamma_v \sigma_t)^2}{2\gamma_d^2 \sigma_t^2}\right) = \alpha$$

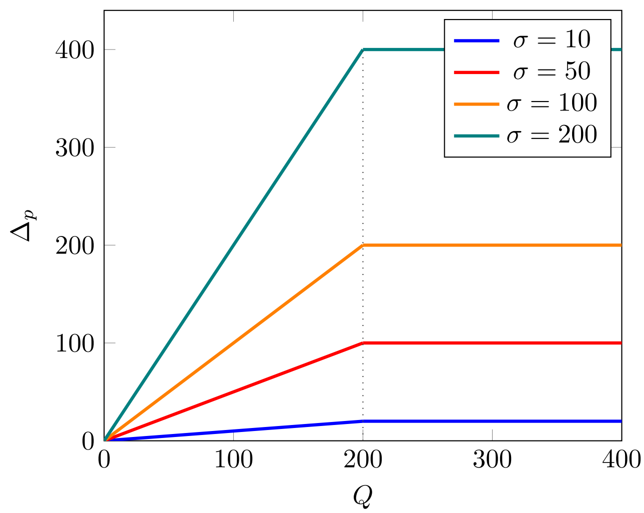

Figure 4: Evolution of Δ_p with Q and σ

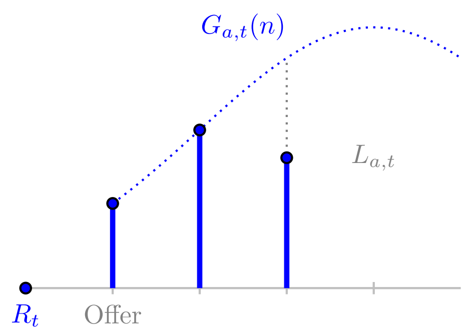

Figure 5: Execution of a market buy order along the liquidity curve

By finding m that satisfies the equation above, we can calculate the execution price of our order. In practice, we use an algorithm that evaluates the value of the liquidity curve at every tick until the order is fully consumed. This gives us m as well as the amount of liquidity consumed at each price tick. Using this, we deduce the order execution price as the average of the prices touched by the order weighted by the size consumed at each price.

Algorithm 1: Calculating execution price for order of size α at $t = \tau$

```
i ← 0;
m ← 0;
α_0 ← α;
while α > 0 do
    β ← min(G_τ(i), α);
    m ← m + β × i;
    α ← α − G_τ(i);
    i ← i + 1;
end
price ← BBO ± (m ÷ α_0);
return price;
```

6 Example

We would like to provide liquidity for the BTC/USD crypto-synthetic market. At $t = 0$ we assume:

- $P_t = \$8,000$, the market mid price
- $\gamma_0 = 0.25$, $\gamma_v = 1$, $\gamma_d = 0.25$, $\gamma_q = 0.2$, $\gamma_r = 0.5$. In practice, all our γs, which are the model's risk aversion and scaling factors, are derived from one global risk aversion factor that applies to all markets. However, individual γs can be set to tune the model with more precisely.
- $\phi = \$0.10$, the market tick size
- $\sigma_t = 52$, expressed in ticks
- $\psi_t = 0.25\sigma = 13$, the base spread
- $L_t = \$500,000$, the total initial liquidity
- $G_t(n) = \frac{500000}{0.25 \cdot 52 \cdot \sqrt{2\pi}} \exp\left(-(\frac{(n-52)^2}{2(0.25 \cdot 52)^2}\right) = \frac{5 \cdot 10^5}{13\sqrt{2\pi}} \exp\left(-(\frac{n-52}{13})^2\right)$, the liquidity curve without volatility or inventory impact
- $R_t = P_t$, the reservation price without inventory skew

Let's assume a trader opens a \$100,000 long position at $t = t_1$. Our inventory is now:

$$Q_{t=t_1} = -100,000$$

and

$$\Delta_q(t_1) = 0.25 \cdot 100,000 = \$25,000$$

Liquidity offered on the ask side is now

$$L_{a,t=t_1} = \$500,000 - \$25,000 = \$475,000$$

And on the bid side

$$L_{b,t=t_1} = \$500,000 + \$25,000 = \$525,000$$

The reservation price is also affected. Price skew in ticks is

$$\Delta_p(t) = \frac{52 \cdot 100,000 \cdot 0.5}{500,000} = 5$$

So the reservation price is now

$$R_t = 8000 + 5 \cdot \phi = \$8000.50$$

7 Conclusion

In this paper, we have described an automated market maker that can provide liquidity to traders on any asset with a price feed. It is zero-intelligence, hence conflict-free. It uses liquidity curves quoted around a reservation price with the goal of providing superior order execution quality while minimizing price and non-execution risk to liquidity providers. We describe the impact of volatility and inventory on the size and distribution of liquidity to obtain a general form liquidity curve formula. We study the impact of inventory imbalance on market maker quotes and obtain a formula for the reservation price, which is the mid price plus an inventory dependent skew. We finally describe algorithms to execute market orders on the liquidity curves, providing order book quality execution without the inefficiencies.

References

[1] Abe Osten. Cap: Crypto-Synthetic Trading. 2020.

[2] Olivier Guéant. Optimal market making. 2017.

[3] SASHA STOIKOV MARCO AVELLANEDA. High-frequency trading in a limit order book. 2007.